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AN ENTROPIC SOLUTION TO THE PARTICLE CORRESPONDENCE PROBLEM

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An automatic tracking algorithm for particle-image velocimetry (PIV) has been developed that satisfactorily tracks 1000 or more identical particles. Time and storage requirements are modest, with a CPU time requirement of 7 sec for 100 particles and 300 sec for 1000 particles on the CRAY 2. The approach works from an artificial pairings matrix V_{ij} that defines the identification of particle i in picture 1 with j in 2 by a value V_{ij} = 1. An entropy-modified distance measure of total particle movement is minimized by a standard Newton-Raphson technique. Values V_{ij} are made to go to either 0 or 1 by the approach. As values V_{ij} approach 1 they are removed from the problem, reducing problem size and CPU time requirements.

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NOMENCLATURE

A	area
D	distance, objective function
E	Eq. (4)
F	Eq. (6)
F	Eq. (7)
i	index
j	index
L_m	mean distance between particles
N	number of particles
P_k	permutation of the index array
Δr_{ij}	distance moved
Δt	time between frames
V_{ij}	matrix
v_m	mean velocity of particles
λ_i	Lagrange multiplier
μ_j	Lagrange multiplier
ρ	weight factor

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1. INTRODUCTION

Particle tracking, as in particle image velocimetry (PIV), requires the identification of each particle in one picture with another in a second picture. The second picture shows the particles after a short amount of time has elapsed so that the particles have moved from their positions in the first picture. The identification problem is particularly difficult when the particles must be regarded as indistinguishable, i.e., individual shapes or intensities cannot be used as reliable cues of identification. A further difficulty arises when a history of particle trajectories, i.e. many pictures of the particle field, is not available as further tracking information. The tracking algorithm described below was developed for specifically these circumstances. A summary of past attempts at solving this problem is given in Ref. 1 and 2.

2. THE TRACKING PROBLEM

2.1 Minimum Movement Criterion

Let there be N particles present in each of pictures 1 and 2. Let $|\Delta r|_{ij}$ represent the inferred distance of movement due to identifying particle i in picture 1 with particle j in picture 2. It is assumed that both pictures have been digitized so that all possible particle movements $|\Delta r|_{ij}$ are known and tabulated. For any set of pairings i, j(i) chosen, there is an inferred total distance of movement

$$D = \sum_{i=1}^{N} |\Delta r|_{ij}. \tag{1}$$

Since the particles are assumed indistinguishable, and assumimg a relatively short time duration between the two pictures, it is logical to choose a pairing rule j(i) such that the total inferred movement of the particles is a minimum.

$$D = \min_{P_k} \sum_{i=1}^{N} |\Delta r|_{ij(i)} . \tag{2}$$

This problem, by its nature, is discrete. The number j to associate with each i must be an integer lying between 1 and N. Therefore, the minimization to be accomplished in (2) does not, at least outwardly, lend itself to solution by a continuous minimum seeking algorithm such as Newton-Raphson. On the other hand, the Newton-Raphson technique is very powerful and easily programmed. For these reasons, it would be nice to convert our discrete problem to an equivalent continuous one.

2.2 Continuous Formulation

A method for accomplishing this is as follows. Introduce a matrix V_{ij} which, temporarily, represents for each particle i a fractional or partial association with each particle j. Thus, each element V_{ij} is selected from the continuum of values between 0 and 1, such that

$$0 \leq V_{ij} \leq 1, \tag{3}$$

Now the total distance D may be replaced by [compare with (2)]

$$E = \sum_i \sum_j V_{ij} \mid \Delta r \mid_{ij}. \tag{4}$$

(All sums go from 1 to N, unless otherwise indicated.) Of course in the final analysis, for each i all V_{ij} should be zero except for one, corresponding to the choice j(i), which will have value unity. However, with a minimization technique, such as Newton-Raphson, with continuously varying parameters, it would be permissible to allow values V_{ij} away from target values 0 or 1, provided they converge in some limiting sense to 0 or 1 eventually. This is the philosophy behind the approach described next.

The Newton-Raphson technique lends itself nicely to equality constraints. The loose requirements (3) may be further refined to a useful set of equalities.

$$\begin{aligned} &\sum_j^N V_{ij} = 1, \quad i = 1, ..., N, \\ &\sum_i^N V_{ij} = 1, \quad j = 1, ..., N-1. \end{aligned} \tag{5}$$

The problem is to minimize E subject to the constraints (5). Introducing Lagrange multipliers λ_i and μ_i we obtain a modified objective function,

$$F = \sum_i \sum_j V_{ij} \mid \Delta r \mid_{ij} + \sum_i \lambda_i \left(\sum_j V_{ij} - 1 \right) + \sum_j \mu_j \left(\sum_i V_{ij} - 1 \right) = \text{minimum} \quad (6)$$

for unconstrained minimization with respect to V_{ij}, λ_i and μ_i.

However, there is one problem with this approach. There is nothing in it that forces values V_{ij} toward either 0 or 1. There is, it turns out, a rather simple way to accomplish this.

2.3 An Entropic Approach

An image restoration technique was introduced some years ago, (Ref. 3) that confines its outputs to lie between 0 and 1. It does this by adding an entropy sum, plus a "complementary entropy" sum, to the quantity that is to be minimized. We can do the same here. The new objective function is

$$G = \sum_i^N \sum_j^N V_{ij} \mid \Delta r \mid_{ij} + \sum_i^N \lambda_i \left(\sum_j^N V_{ij} - 1 \right) + \sum_j^{N-1} \mu_j \left(\sum_i^N V_{ij} - 1 \right)$$

$$+\rho \sum_i^N \sum_j^N [V_{ij} \ln V_{ij} + (1 - V_{ij}) \ln(1 - V_{ij})] \,. \quad (7)$$

The last double sum consists of entropy VlnV terms and "complementary entropy" (1 - V) ln (1 - V) terms. Weight factor ρ is at the user's disposal and is taken up below.

We now seek the calculus-extremum solution to (7), and see if it can be forced toward 0 or 1, as claimed. Setting $\partial / \partial V_{ij} = 0$ of Eq. (7) yields the intriguing result

$$V_{ij} = \frac{1}{1 + \exp\left[(1/\rho) (\mid \Delta r \mid_{ij} + \lambda_i + \mu_j) \right]} , \quad i, j = 1,\ldots N. \quad (8)$$

The action of weight ρ is now apparent. If we make ρ very small then the exponent tends toward either $+\infty$ or $-\infty$ forcing V_{ij} toward either 0 or 1, just as we want.

There are four double sums, or terms, in G, Eq. (7). The size of ρ determines the relative weight of the last term for the minimization. Because we want all the emphasis to be upon the first terms our objective function F, Eq. (6), we want ρ to approach zero.

The particulars of the Newton-Raphson approach that solves the problem (7), (8) are as follows.

3. Newton-Raphson Tracking Algorithm

The aim is to arrive at a set of parameters $\{\lambda_i\}$, $\{\mu_j\}$ such that V_{ij} given by representation (8) satisfies the equality constraints (5). This is accomplished as follows.

(a) Fix weight ρ, e.g., $\rho = 10$. ρ must be positive.

(b) Start with a trial solution $\{\lambda_i\}$, $\{\mu_j\}$.

(c) Form V_{ij} by Eq. (8), all i,j.

(d) Form sums $t_i = \sum_j V_{ij}$, $u_j = \sum_i V_{ij}$, all i,j

(e) If all t_i and all u_j equal 1, STOP.

(f) If not, set $\Delta t_i = 1 - t_i$, $\Delta u_j = 1 - u_j$, all i, j. (9)

(g) Solve for changes $\Delta\lambda_i$, $\Delta\mu_j$, all i, j, in 2N equations

$$\Delta t_i = \frac{1}{\rho} \sum_j V_{ij}(1 - V_{ij})\,(\Delta\lambda_i + \Delta\mu_j)$$

$$\Delta u_j = \frac{1}{\rho} \sum_i V_{ij}(1 - V_{ij})\,(\Delta\lambda_i + \Delta\mu_j)$$

The coefficients of $\{\Delta\lambda_i\}$, $\{\Delta\mu_j\}$ comprise matrix (11) below

(h) Update $\lambda_i \rightarrow \lambda_i + \Delta\lambda_i$, $\mu_j \rightarrow \mu_j + \Delta\mu_j$

(i) Go to (c).

Once the loop STOPS , at step (e), weight parameter ρ is reduced by a factor 1/2, and step (c) starts a new Newton-Raphson problem, using a previous solution set $\{\lambda_i\}$, $\{\mu_j\}$ as the new trial solution. Weight factor ρ is reduced, problem by problem, until all V_{ij} values are sufficiently close to either 0 or 1. This solves the correspondence problem (2).

3.1 Accuracy

Can we be assured that a stationary solution to (7) is a minimum, and in particular, the global minimum? To judge this effect, take the second partial derivative of form (7). This yields a quantity

$$\rho[1/V_{ij} + 1/(1 - V_{ij})] \tag{10}$$

Since V_{ij} must be positive, by representation (8), and also ρ is positive, quantity (10) must be positive. Therefore function (7) is concave downwards, and there is only one stationary solution, a minimum. Hence, the output of algorithm (9) must always attain the absolute minimum to the problem.

This was confirmed by experimentation with the algorithm. For small dimensioned problems, where N $\leq$ 8, the absolute minimum solution can be obtained by simply trying out all possible particle pairings. In all cases tried, where particle positions were generated with uniform randomness, the solution obtained by algorithm (9) matched that by simple permutation. A case N = 20 is shown in Fig. 1.

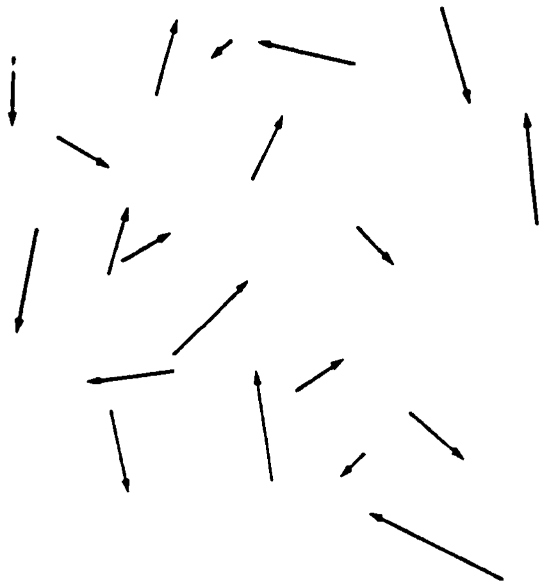

Figure 1. A case of N=20 particle pairs. Arrows designate connections established by algorithm (9).

3.2 Efficiency

The objective is to use the algorithm on very large problems, where N is the order of 1000 or more particles. On the other hand, step (g) of algorithm (9) requires inversion of a 2N x 2N matrix, and this is done repeatedly during enaction of the algorithm. To avoid the requirement for multiple inversion of 2000 x 2000 (and more) matrices, two tactics were taken. These dramatically speed up execution of the algorithm.

(1) The V_{ij} values that approach 1 do so at different rates. Some do after only 3 or 4 iterations, while others require 40 or more. Also, once a V_{ij} is close to 1, it rarely backs up and approaches 1 ultimately. Hence, to avoid the need for carrying along this unnecessary baggage, the algorithm removes from the problem any i, j pair whose V is close enough to 1, and then continues the problem with the remaining elements. In this way, the problem rapidly reduces in dimensionality after but a few iterations. For larger problems of $N \geq 50$, the saving in CPU time due to this tactic is greater than 90%.

(2) The inversion subroutine, that is used in step (g) contains three nested DO loops. This makes it the major user of time for the algorithm. However, the matrix that is inverted has the special form

$$
\begin{bmatrix}
\sum_j W_{1j} & 0 & \cdot\cdot & 0 & W_{11} & W_{12} & \cdots & W_{1N} \\
0 & \sum_j W_{2j} & & 0 & W_{21} & W_{22} & \cdots & W_{2N} \\
0 & 0 & & 0 & \cdot & & & \cdot \\
\vdots & \vdots & & \vdots & \vdots & & & \vdots \\
0 & 0 & \cdot\cdot & \sum_j W_{Nj} & W_{N1} & W_{N2} & \cdots & W_{NN} \\
W_{11} & W_{21} & \cdots & W_{N1} & \sum_j W_{j1} & 0 & \cdots & 0 \\
W_{12} & W_{22} & \cdots & W_{N2} & 0 & \sum_j W_{j2} & \cdots & 0 \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
W_{1N} & W_{2N} & \cdots & W_{NN} & 0 & 0 & \cdots & \sum_j W_{jN}
\end{bmatrix}
\qquad (11)
$$

where $W_{ij} \equiv V_{ij}(1 - V_{ij})$. Since all V_{ij} are positive, the diagonal elements of this matrix dominate.

On the other hand, the innermost DO loop operates on off-diagonal elements. Hence, it was possible to replace this DO by a single operation, effectively changing the three nested DO's to two. It should be emphasized that the result is now only an approximate inversion of the matrix. However, since the Newton-Raphson algorithm is iterative anyhow, it can recover from the inaccuracy. In practice, it does: the method reduces execution time by about 30 to 40%.

Another opportunity for time saving grows out of the off diagonal zeros in the upper left hand quadrant of (11). This permits the inner DO loop to be contracted by about a factor of 25%, and without further loss of accuracy in the inversion.

3.3 Empirical Results

We generated random particle configurations for the two frames, and then processed them by use of the tracking algorithm (9). With up to N = 8 particles in each picture, results were confirmed by permutations-algorithm that searches for the absolute minimum in D by exhaustive search of all solution space. For values of N greater than 8 quantitative assessment of the method is possible based on computer experiments, such as simulated Brownian motion. There, both the initial and final position of particles are known and the accuracy of the matching by the algorithm can be quantified.

Three cases, consisting of 100, 250 and 1000 particles each, see Figs. 2, 3 and 4, were run. The particles were displaced by amounts determined by a random number generator and the algorithm was used to attempt matching of the particles. The results, in CPU seconds on a Cray-2 running UNICOS 5.0, may be summarized as follows:

TABLE 1.

No of particles	Case 1	Case 2	Case 3**
100	4.6	4.1	8.2
250	43.5	27.6	31.5
1000	1329.0*	283.7	237.3

* This case did not run to completion. In this length of time the problem had been reduced to 138 particles.

** Sample cases with three different matrix inversion strategies were run. Exact inversion was used in case 1. In case 2 inexact inversion was used until the number of iterations exceeded 9. Case 3 employed inexact inversion. In each case three samples consisting of 100, 250 and 1000 particles were analyzed. The results are summarized in Table 1 showing the CPU seconds utilized.

The memory requirements for the test cases that were run on the Cray-2 are:

TABLE 2.

100 particles:	158,345 words***
250 particles:	582,321 words
1000 particles:	8,102,321 words

*** Each word consists of 64 bits (8 bytes).

For reasonable expectation for a solution to the identification problem, several conditions need to be met:

(1). In their initial distribution, in the plane of observation, the particles must not be packed too closely. In situations where particles are in close proximity and where there is not a well defined flow direction, such as for example in Brownian motion, crossing of trajectories may occur, possibly leading to erroneous selection of terminal positions. (2). Condition 1 implies that the maximum allowable displacement between frames should be limited. It has been found that

$$\Delta t \mid v_m \mid < 0.5\, L_m \tag{12}$$

is a reasonable choice. Here, Δt is the time elapsed between frames and L_m the mean distance between particles in the initial distribution and v_m the mean velocity of the particles. In general, the mean distance between particles in a plane can be expressed as

$$L_m = \sqrt{4A/\pi N} \tag{13}$$

where L_m is the mean distance and A is the area, N the number of particles.

(3). Finally, it must be ascertained that there is no temporal variation in the number of particle between frames. This ensures that pairing of all the particles is possible.

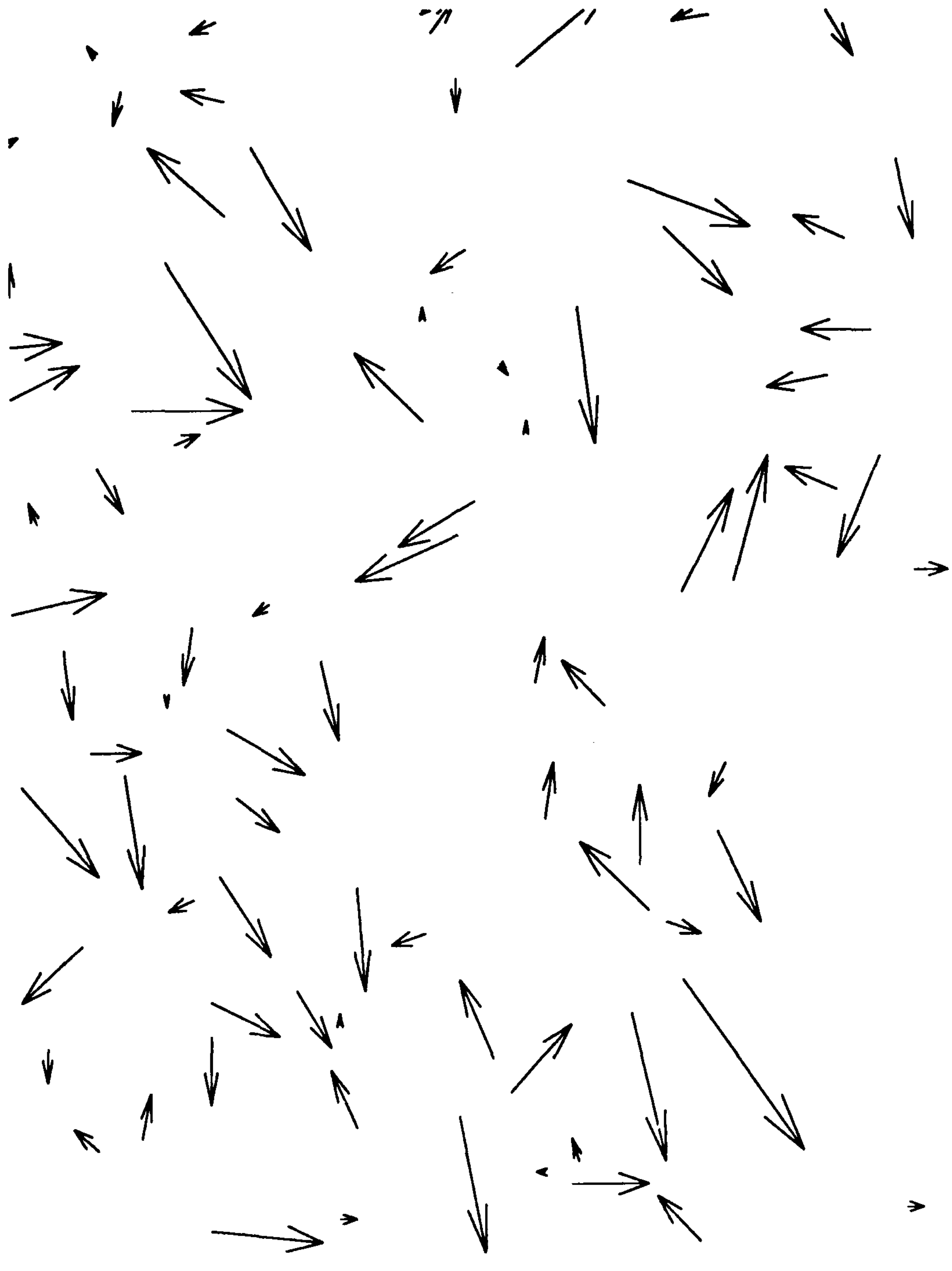

Figure 2. A case with N = 100 particles. Arrows indicate established connections between particles in the two frames.

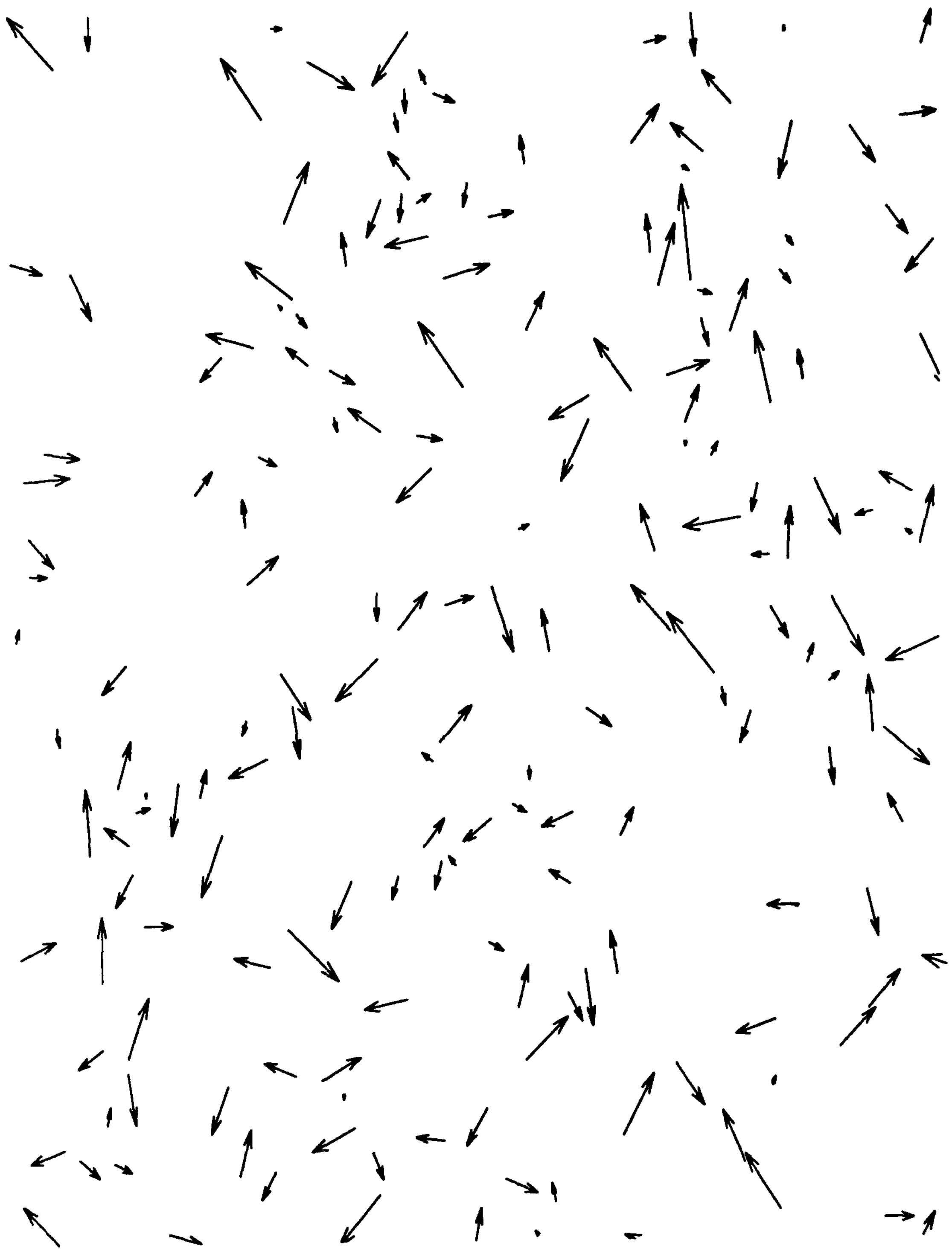

Figure 3. The matchup of 250 particle pairs. Arrows indicate matched particles.

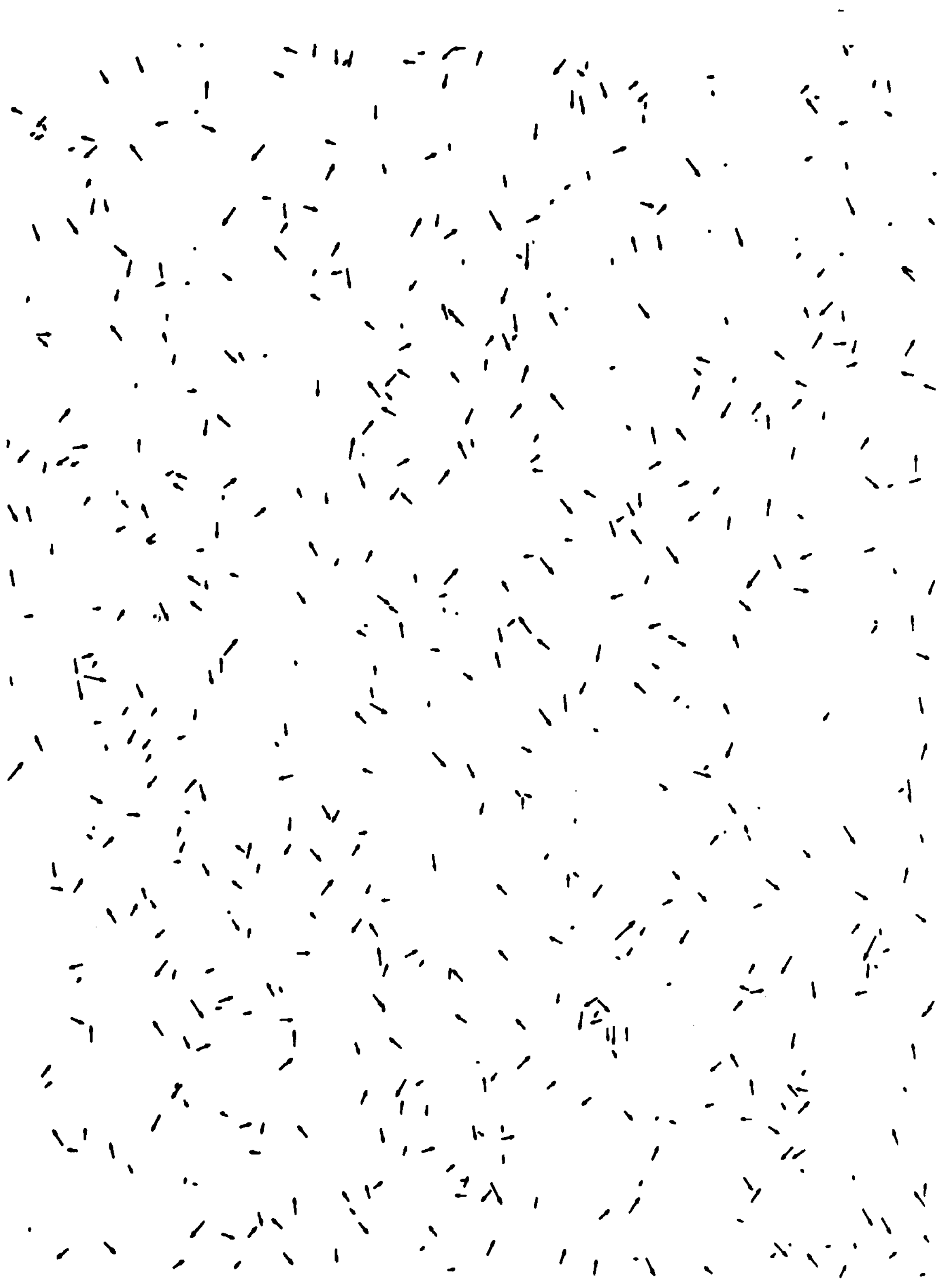

Figure 4. The case with N = 1000 particles. Arrows indicate matched particle pairs.

4. CONCLUSIONS

A tracking algorithm has been developed that automatically minimizes total distance of movement as a criterion of identification of particle pairs. The algorithm always produced the absolute minimum distance-pairings as its solution. Cases of up to 1000 particle pairs were run successfully, and with small CPU time requirement. The algorithm is unsuitable for situations where the volume particle density is high. Effort is under way to extend the applicability of this methodology to such cases.

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